PS-003-1164002

Seat No. ______

M. Sc. (Mathematics) (Sem. IV) Examination

August - 2020

CMT-4002 : Integration Theory

Faculty Code : 003
Subject Code : 1164002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Each question carry 14 marks.
(3) Figures on the right indicates marks.

1 Answer any seven questions : **7×2=14**

(1) Give only statements of Fatou's lemma.

(2) Define the simple function and also write its canonical representation.

(3) State only the statement of Radon-Nikydom Theorem for signed measure space.

(4) Give only statement of Fubini's Theorem.

(S) Write the statement of Uryson's lemma.

(6) Write the definition of Borel σ-algebra on $\mathbb{R}$.

(7) Write the definition of locally compact space.

(8) Define the measurable function also write one example of measurable function.

(9) Write the definition of positive set and negative set for a signed measure space.

(10) Define the word saturated measure space and also write one example of saturated space.

2 Answer any **two** questions : **2×7=14**

(a) Let f and g be nonnegative measurable function on a measurable space (X, A, μ) then $\int f + gd\mu = \int fd\mu + \int gd\mu$ for every measurable subset E of X.

(b) Let $\mu_1, \mu_2, ..., \mu_k$ be measure on (X, A) and let $\alpha_1, \alpha_2, ..., \alpha_k$ be nonnegative real numbers then $\alpha_1\mu_1 + \alpha_2\mu_2 + ... + \alpha_k\mu_k$ is a measure on (X, A).

(c) State and Prove the Lebesgue Dominated Convergence Theorem.

3 Answer the following both. **14**

(a) State and Prove Hahn-Decomposition also if X is any nonempty set and $v = \delta_{x_0} - \eta$ defined on $P(X)$, Where $x_0 \in X$ and η is the counting measure, then find Hahn-Decomposition of v.

(b) State and Prove Monotone Convergence Theorem.

OR

3 Answer the following both : **14**

(a) Let (X, A, μ) be a σ-finite measure space and let v be a finite signed measure on (X, A) that is absolutely continuous with respect to μ then show that there is an integrable function f on X (with respect to μ) such that $v(E) = \int_E fd\mu$ for every $E \in A$.

(b) State and prove Jordan Decomposition and also prove the Uniqueness of Jordan Decomposition of signed measure.

4 Answer any two questions : **2×7=14**

(a) Define Measure absolutely continuous with respect to another measure and mutually singular measure also show that if (*X*, *A*) is a measure space and v and μ be a signed measure on (*X*, *A*) if $v \perp \mu$ and $v << \mu$ then prove that $\mu = 0$.

(b) State without proof Fubini's Theorem and Tonelli's Theorem.

(c) Let *X* be a Locally compact separable metric space then prove that $B_0(X) = B_a(X)$.

(d) Let μ^* be an outer measure on *X* and let $\{E_n\}$ be a sequence of pair wise disjoint measurable subset of *X* then show that $\Sigma_n \mu^* (A \cap E_n) = \mu^* (A \cap (\cup_n E_n))$.

5 Answer any **two** of the following questions : **2×7=14**

(a) Let μ^* be an outer measure on *X* then show that collection *B* of all μ^* measurable subset of *X* is a σ-algebra.

(b) Define the word Locally compact Hausdorff space also show that if *X* is locally compact Hausdorff space then $B_a(X) \subset B_0(X)$.

(c) State and Prove The Simple Approximation Theorem.

(d) State without proof Caratheodary extension theorem Give an example to show that σ-finite assumption in the theorem can not be dropped.